Objectives

Reading assignment review

Introduction to Algorithms

- Our process, through an example

Representative problems

Assignment: Review Paper

Read paper

- 2 hours max
- Don't get bogged down in terminology
- Addendum:
  - Benefits of approach
  - Results of approach

Review paper

- Almost an abstract
- Write on Sakai forum
- Due by 10 a.m. on Friday

Friday: Discuss paper and questions

Details

Sakai: Log in

Meeting with Candidates?

CS Talks

Attend two CS talks, all in Parmly 405

- Mon, Jan 12, D period
  - Andrea Tartaro: "Authorable Virtual Peers: Using Computer Science to Understand and Support Children with Special Needs"
- Thurs, Jan 15, 3:30 p.m.
  - Mark Liffiton, "Satisfying Constraints, and What To Do When You Can't"
- Fri, Jan 23, 4 p.m.
  - Joshua Stough, "Appearance Models for Medical Image Segmentation"

Post summary on Sakai

- More on that later

ALGORITHMS

Computational Problem Solving 101

Computational Problem

- A problem that can be solved by logic

To solve the problem:

- Create a **model** of the problem
- Design an **algorithm** for solving the problem using the model
- Write a **program** that *implements* the algorithm

Computational Problem Solving 101

Algorithm: a well-defined recipe for solving a problem

- Has a finite number of steps
- Completes in a finite amount of time

Program

- An algorithm written in a **programming language**
- Also called code

Application

- Large programs, solving many problems

Addendum

Before creating algorithm, define the problem

- Simplify as necessary

Matching Residents to Hospitals

Goal: Given a set of preferences among hospitals and medical school students, design a self-reinforcing admissions process.

Unstable pair: applicant *x* and hospital *y* are unstable if:

- *x* prefers *y* to its assigned hospital
- *y* prefers *x* to one of its admitted students

Stable assignment: Assignment with no unstable pairs

- Natural and desirable condition
- Individual self-interest will prevent any applicant/hospital deal from being made

What details make this problem tricky?

Stable Matching Problem

Simplified version of resident-matching problem

Goal: Given *n* men and *n* women, find a "suitable" matching

- Participants rate members of opposite sex
- Each man lists women in order of preference from best to worst
- Each woman lists men in order of preference from best to worst

favorite ↓ … least favorite ↓

	1st	2nd	3rd
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

Men's Preference Profile

favorite ↓ … least favorite ↓

	1st	2nd	3rd
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

Women's Preference Profile

Stable Matching Problem

Perfect matching: everyone is matched monogamously

- Each man is paired with exactly one woman
- Each woman is paired with exactly one man

Stability: no incentive for some pair of participants to undermine assignment by joint action

- In matching M, an *unmatched* pair m-w is unstable if man *m* and woman *w* prefer each other to current partners
- Unstable pair m-w could each improve by eloping

Stable matching: perfect matching with no unstable pairs

Stable matching problem. Given the preference lists of *n* men and *n* women, find a stable matching if one exists.

Analyzing Stability

Q. Is assignment X-C, Y-B, Z-A stable?

- Unstable: m prefers w to his woman; w prefers m to her man

favorite ↓ … least favorite ↓

	1st	2nd	3rd
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

Men's Preference Profile

favorite ↓ … least favorite ↓

	1st	2nd	3rd
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

Women's Preference Profile

Stable Matching Problem

Q. Is assignment X-C, Y-B, Z-A stable?

A. No. Bertha and Xavier prefer each other

favorite ↓ … least favorite ↓

	1st	2nd	3rd
Xavier	Amy	Bertha	Clare
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Men's Preference Profile

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Women's Preference Profile

13

Stable Matching Problem

Q. Is assignment X-A, Y-B, Z-C stable?

A. Yes.

favorite ↓ … least favorite ↓

	1st	2nd	3rd
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

Men's Preference Profile

favorite ↓ … least favorite ↓

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Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

Women's Preference Profile

14

Any Questions?

What are you wondering about this problem at this point?

Jan 7, 2009 CS211 15

Propose-And-Reject Algorithm

[Gale-Shapley 1962]

Intuitive method that guarantees to find a stable matching

```
Initialize each person to be free.
while (some man is free and hasn't proposed to every woman) {
    Choose such a man m
    w = 1st woman on m's list to whom m has not yet proposed
    if (w is free)
        assign m and w to be engaged
    else if (w prefers m to her fiancé m')
        assign m and w to be engaged, and m' to be free
    else
        w rejects m
}
```

16

Applying the Algorithm

	1st	2nd	3rd
Xavier	Amy	Bertha	Clare
Yancey	Bertha	Amy	Clare
Zeus	Amy	Bertha	Clare

Men's Preference Profile

	1st	2nd	3rd
Amy	Yancey	Xavier	Zeus
Bertha	Xavier	Yancey	Zeus
Clare	Xavier	Yancey	Zeus

Women's Preference Profile

```
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        assign m and w to be engaged, and m' to be free
    else
        w rejects m
}
```

17

Observations about the Algorithm

What can we say about any woman's partner over the execution of the algorithm?

How does a woman's state change over the execution of the algorithm?

What can we say about a man's partner?

Jan 7, 2009 CS211 18

Algorithm Analysis

What is the running time of this algorithm?

What is the state complexity of this algorithm?

Proof of Correctness: Termination

Observation 1. Men propose to women in decreasing order of preference

Observation 2. Once a woman is matched, she never becomes unmatched; she only "trades up"

Claim. Algorithm terminates after at most n^2 iterations of while loop.

	1st	2nd	3rd	4th	5th
Victor	A	B	C	D	E
Wyatt	B	C	D	A	E
Xavier	C	D	A	B	E
Yancey	D	A	B	C	E
Zeus	A	B	C	D	E

	1st	2nd	3rd	4th	5th
Amy	W	X	Y	Z	V
Bertha	X	Y	Z	V	W
Clare	Y	Z	V	W	X
Diane	Z	V	W	X	Y
Erika	V	W	X	Y	Z

$n(n-1) + 1$ proposals required

Proof of Correctness: Termination

Observation 1. Men propose to women in decreasing order of preference

Observation 2. Once a woman is matched, she never becomes unmatched; she only "trades up"

Claim. Algorithm terminates after at most n^2 iterations of while loop.

Pf. Each time through the while loop a man proposes to a new woman. There are only n^2 possible proposals.

$n(n-1) + 1$ proposals required

Assignments

Read Tartaro paper

- Discuss on Friday

Read Chapter 1 of Algorithms book

- Return to next Friday