

CS211: Problem Set 2

Due Friday, February 13

1. (3.1) Consider the directed acyclic graph G in Figure 3.10 on page 107 of the book. How many topological orderings does it have? (Hint: Review solved Exercise 1)
2. (3.9). There's a natural intuition that two nodes that are far apart in a communication network, i.e., separated by many hops, have a more tenuous connection than two nodes that are close together. There are a number of algorithmic results that are based to some extent on different ways of making this notion precise. Here's one that involves the susceptibility of paths to the deletion of nodes.

Suppose that an n -node undirected graph $G = (V, E)$ contains two nodes s and t such that the distance between s and t is strictly greater than $n/2$. Show that there must exist some node v , not equal to either s or t , such that deleting v from G destroys all $s-t$ paths. (In other words, the graph obtained from G by deleting v contains no path from s to t .) Give an algorithm with running time $O(m+n)$ to find such a node v .

3. (3.12). You're helping a group of ethnographers analyze some oral history data they've collected by interviewing members of a village to learn about the lives of people who've lived there over the past two hundred years.

From these interviews, they've learned about a set of n people (all of them now deceased), whom we'll denote P_1, P_2, \dots, P_n . They've also collected facts about when these people lived relative to one another. Each fact has one of the following two forms:

- For some i and j , person P_i died before person P_j was born; or
- for some i and j , the life spans of P_i and P_j overlapped at least partially.

Naturally, they're not sure that all these facts are correct; memories are not so good, and a lot of this was passed down by word of mouth. So what they'd like you to determine is whether the data they've collected is at least internally consistent, in the sense that there could have existed a set of people for which all the facts they've learned simultaneously hold.

Give an efficient algorithm to do this: either it should produce proposed dates of birth and death for each of the n people so that all the facts hold true, or it should report that no such dates can exist—that is, the facts collected by the ethnographers are not internally consistent. (Hint: Problem 3.3 may be helpful.)