

# CS211: Problem Set 1

Due Friday, January 22  
Points Possible: 25

1. In a room with  $n$  people ( $n > 2$ ), every person shakes hands once with every other person. Prove that there are  $\frac{n^2-n}{2}$  handshakes.
2. (2.1-8, CLR) We can extend the  $O$  notation to the case of two parameters  $n$  and  $m$  that can go to infinity independently at different rates. For a given function  $g(n, m)$ , we denote  $O(g(n, m))$  as the set of functions

$O(g(n, m)) = \{f(n, m): \text{there exist positive constants } c, n_0, m_0 \text{ such that } 0 \leq f(n, m) \leq cg(n, m) \text{ for all } n \geq n_0, m \geq m_0\}$

Give corresponding definitions for  $\Omega(g(n, m))$  and  $\Theta(g(n, m))$ .

3. (2.3) Take the following list of functions and arrange them in ascending order of growth rate. That is, if function  $g(n)$  immediately follows function  $f(n)$  in your list, then it should be the case that  $f(n)$  is  $O(g(n))$ .

$$\begin{array}{ll} f_1(n) = n^{2.5} & f_2(n) = \sqrt{2n} \\ f_3(n) = n + 10 & f_4(n) = 10^n \\ f_5(n) = 100^n & f_6(n) = n^2 \log n \end{array}$$

4. (2.4) Take the following list of functions and arrange them in ascending order of growth rate. That is, if function  $g(n)$  immediately follows function  $f(n)$  in your list, then it should be the case that  $f(n)$  is  $O(g(n))$ .

$$\begin{array}{ll} f_1(n) = 2^{\sqrt{\log n}} & f_2(n) = 2^n \\ f_3(n) = n^{4/3} & f_4(n) = n(\log n)^3 \\ f_5(n) = n^{\log n} & f_6(n) = 2^{2^n} \\ f_7(n) = 2^{n^2} & \end{array}$$

5. Suppose that each row of an  $n \times n$  array  $A$  consists of 1's and 0's such that, in any row  $i$  of  $A$ , all the 1's come before any 0's in that row. Suppose further that the number of 1's in row  $i$  is at least the number in row  $i + 1$ , for  $i = 0, 1, \dots, n - 2$ . Assuming  $A$  is already in memory, describe a method running in  $O(n)$  time for counting the number of 1's in the array.