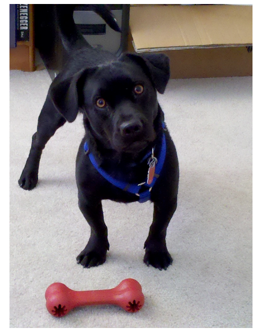
Objectives

- Introduction to Algorithms, Analysis
- Course summary
- Reviewing proof techniques

What I Did On My Leave

- Research
- Coding
- Writing



What This Course Is About

From *30 Rock*

Now, everything comes down to expert knowledge of **algorithms** and **data structures**.
If you don't speak fluent **O-notation**, you may have trouble getting your next job at the technology companies in the forefront.
-- Larry Freeman

For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense, and even mysterious. But once unlocked, they cast a **brilliant new light** on some aspect of computing.
-- *Francis Sullivan*

What is an Algorithm?

- Precise procedure to solve a problem
- Completes in a finite number of steps

Questions to Consider

- What are our goals when designing algorithms?
- How do we know when we've met our goals?

- Goals: Correctness, Efficiency
- Use analysis to show/prove

Course Goals

- Learn how to formulate precise problem descriptions
- Learn specific algorithm design techniques and how to apply them
- Learn how to analyze algorithms for efficiency and for correctness
- Learn when no exact, efficient solution is possible

Course Content

- Algorithm analysis
  - Formal – proofs; Asymptotic bounds
- Advanced data structures, e.g., heaps, graphs
- Greedy Algorithms
- Dynamic Programming
- Divide and Conquer
- Network Flow
- Computational Intractability

Course Notes

- Textbook: *Algorithm Design*
  - Optional: CLRS
- Participation is encouraged
  - Individual, group, class
- Assignments:
  - Reading text, writing brief summaries
    - Readings through Friday due following Wednesday
  - Solutions to problems
  - Analysis of solutions
  - Programming (little)

Given on Friday, due next Friday

Course Grading

- 40% Individual written and programming homework assignments
- 10% Text book reading summaries, weekly
  - In a journal on wiki
- 25% Midterms
- 20% Final
- 5% Participation and attendance

Journal Content

- Brief summary of chapter/section
  - ~1 paragraph of about 5 sentences/section; feel free to write more if that will help you
- Include motivations for the given problem, as appropriate
- Questions you have about motivation/solution/proofs/analysis
- Discuss anything that makes more sense after reading it again, after it was presented in class (or vice versa)
- Anything that you want to remember, anything that will help you
- Say something about how readable/interesting the section was on scale of 1 to 10

ALGORITHMS

Computational Problem Solving 101

- Computational Problem
  - A problem that can be solved by logic
- To solve the problem:
  1. Create a *model* of the problem
  2. Design an *algorithm* for solving the problem using the model
  3. Write a *program* that implements the algorithm

Computational Problem Solving 101

- Algorithm: a well-defined recipe for solving a problem
  - Has a finite number of steps
  - Completes in a finite amount of time
- Program
  - An algorithm written in a programming language

PROOFS

Why Proofs?

- What are insufficient alternatives?
- How can we prove something isn't true?

Why Proofs?

- What are insufficient alternatives?
  - Examples
    - Considered all possible?
  - Empirical/statistical evidence
    - Ex: "Lying" with statistics
- How can we prove something isn't true?
  - One counterexample

Need irrefutable proof that something is true—for ***all*** possibilities

From Joel Feinstein
University of Nottingham
"Why do we do proofs"

Analyzing Statistics

Two hospitals (A and B) each claim to be better at treating a certain disease than the other.

Hospital A
- cured a greater % of its *male patients* last year than Hospital B
- cured a greater % of its *female patients* last year than Hospital B

Hospital B
- cured a greater % of its *patients* last year than Hospital A

Given that none of the #s involved are zero, is it possible that both hospitals have their calculations correct?
If so, which hospital would you rather be treated by?

Example

From Joel Feinstein
University of Nottingham
"Why do we do proofs"

Hospital	Male Patients	%	Female Patients	%	Total Patients	%
A	50/100	50%	1/1	100%	51/101	50.5%
B	24/50	48%	49/50	98%	73/100	73%

Well-known phenomenon: Simpson's Paradox

Common Types of Proofs?

Common Types of Proofs

- Direct proofs
  - Series of true statements, each implies the next
- Proof by contradiction
- Proof by induction

Proof By Contradiction

What are the steps to a proof by contradiction?

Proof By Contradiction

1. Assume the thing we want to prove is false
2. Reason to a contradiction
3. Conclude that it must therefore be true

Prove: There are Infinitely Many Primes

- What is a prime number?
- What is not-a-prime number?

- What is our first step (proof by contradiction)?
- What do we want to show?

Prove: There are Infinitely Many Primes

- Assume there are only finitely many prime numbers
  - List them: p_1, p_2 ..., p_n
- Consider the number $q = p_1p_2... p_n + 1$

What are the possibilities for q?

q is either composite or prime

Prove: There are Infinitely Many Primes

- Assume there are only finitely many prime numbers
  - List them: p_1, p_2 ..., p_n
- Consider the number $q = p_1p_2... p_n + 1$
- Case: q is composite
  - If we divide q by any of the primes, we get a remainder of 1 → q is not composite

Prove: There are Infinitely Many Primes

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- Consider the number $q = p_1p_2... p_n + 1$
- Case: q is composite
  - If we divide q by any of the primes, we get a remainder of 1 → q is not composite
- Therefore, q is prime, but q is larger than any of the finitely enumerated prime numbers listed → *Contradiction*

Proof thanks to Euclid

Proof By Induction

What are the steps to a proof by induction?

Proof By Induction

1. What you want to prove
2. Base case
   - Typical: Show statement holds for $n = 0$ or $n = 1$
3. Assumption for n (***induction hypothesis***)
4. Induction step: show that adding one to n also holds true
   - Often relies on earlier assumptions

When/why is induction useful?

Show true for all (infinite) possibilities
Show works for "one more"

Warm Up

Prove:
$2+4+6+8+\ldots + 2n = n*(n+1)$

Proof

Prove: 2+4+6+8+… + 2n = n*(n+1)

- **Base case**: n = 1 → 2*1 = 1*(1+1) ✔
- Assume true for n
- Prove for n+1

Proof

Prove: 2+4+6+8+… + 2n = n*(n+1)

- **Base case**: n = 1 → 2*1 = 1*(1+1) ✔
- Assume true for n
- Prove for n+1
  - 2+4+6+8+… + 2n + 2(n+1)
  - = n*(n+1) + 2(n+1)
  - = n^2 + n + 2n + 1 = n^2 + 3n + 1
  - = (n+1)*(n+2)
  - = (n+1)*((n+1)+1) ✔

I want to see these steps in your proofs!

Proof: All Horses Are The Same Color

- **Base case**: If there is only *one* horse, there is only one color.
- **Induction step**: Assume as induction hypothesis that within any set of n horses, there is only one color.
  - Look at any set of n + 1 horses
  - Label the horses: 1, 2, 3, ..., n, n + 1
  - Consider the sets {1, 2, 3, ..., n} and {2, 3, 4, ..., n + 1}
  - Each is a set of only n horses, therefore within each there is only one color
  - Since the two sets overlap, there must be only one color among all n + 1 horses

Where is the error in the proof?

Error in Proof

- **Base case**: If there is only *one* horse, there is only one color.
- **Induction step**: Assume as induction hypothesis that within any set of n horses, there is only one color.
  - Look at any set of n + 1 horses
  - Number them: 1, 2, 3, ..., n, n + 1
  - Consider the sets {1, 2, 3, ..., n} and {2, 3, 4, ..., n + 1}
  - Each is a set of only n horses, therefore within each there is only one color
  - *Since the two sets overlap*, there must be only one color among all n + 1 horses

Does not hold true when n+1=2

Lesson: check assumptions within proof

Proof Summary

- Need to prove conjectures
- Common types of proofs
  - Direct proofs
  - Contradiction
  - Induction
- Common error: not checking/proving assumptions
  - “Jumps” in logic

For Next Time

- Check out course wiki page
  - Test username/password
  - Decide which style of journal you want: wiki or blog
- Read first two pages of book’s preface, Chapter 1 of book
  - Summarize on Wiki by next Wednesday @ 10 a.m.