Objectives

- Data structure: Heaps
- Implementing a Priority Queue

Review: Priority Queues for Sorting

1. Add elements into PQ with the number's value as its priority
2. Then extract the smallest number until done
   - Come out in sorted order

Sorting *n* numbers takes O(n logn) time, which is our goal running time. However, "known" data structures won't give us that running time.

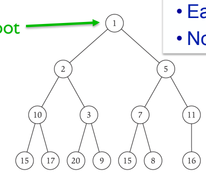
Already know our "loops" will be O(n)

Review: Heap Defined

- Combines benefits of sorted array and list
- Balanced binary tree

- Each node has *at most* 2 children
- Node value is its *key*

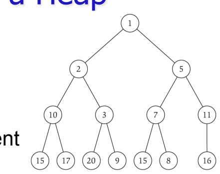
root



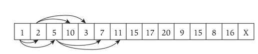
Heap order: each node's key is at least as large as its parent's

Note: **not** a binary search tree

Review: Implementing a Heap



- Option 1: Use pointers
  - Each node keeps
    - Element it stores, key
    - 3 pointers: 2 children, parent
- Option 2: No pointers
  - Requires knowing upper bound on *n*
  - For node at position *i*
    - left child is at *2i*
    - right child is at *2i+1*



Review: Heapify-Up

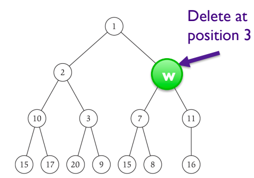
Heap Position where node added

```
Heapify-up(H, i):
  if i > 1 then
      j=parent(i)=floor(i/2)
      if key[H[i]] < key[H[j]] then
        swap array entries H[i] and H[j]
        Heapify-up(H, j)
```

When is this algorithm used?
What is the intuition?
What is the run time?

Deleting an Element

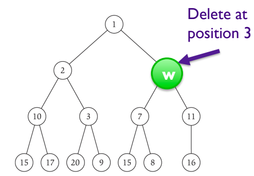
Delete at position 3



Deleting an Element

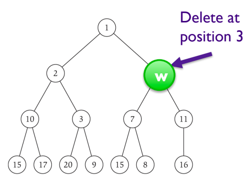
- Delete at position *i*
- Removing an element:
  - Messes up heap order
  - Leaves a "hole" in the heap
- Not as straightforward as `Heapify-Up`
- Algorithm
  1. Fill in element where hole was
     - Patch hole: move n^{th} element into i^{th} spot
  2. Adjust heap to be in order
     - At position *i* because moved n^{th} item up to *i*

Deleting an Element



- What are the possibilities when we move n^{th} element (*w*) into spot where element was removed?

Deleting an Element

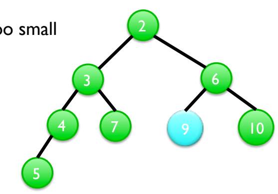


Example of OK:
11 deleted, replaced by 16

- Two "bad" possibilities: element *w* is
  - Too small: violation is between it and parent → `Heapify-Up`
  - Too big: with one or both children → `Heapify-Down` (example: w = 12)

Deleting an Element

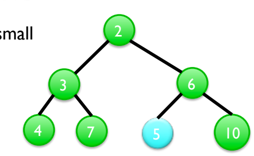
Example where new key is too small



- Delete 9
- Replace with 5

Deleting an Element

Example where new key is too small



- Delete 9
- Replace with 5
- But 5 < 6, so need to `Heapify-Up`

Heapify-Down

```
Heapify-down(H, i):
   n = length(H)
   if 2i > n then
      Terminate with H unchanged
   else if 2i < n then
      left=2i and right=2i+1
      j be index that minimizes
            key[H[left]] and key[[H[right]]
   else if 2i = n then
      j=2i

   if key[H[j]] < key[H[i]] then
      swap array entries H[i] and H[j]
      Heapify-down(H, j)
```

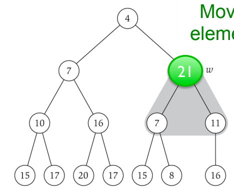
Why can we stop?

Heapify-Down

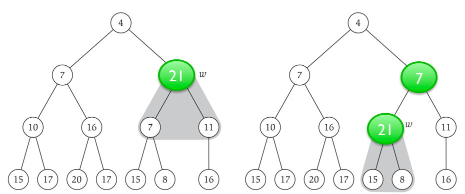
```
Heapify-down(H, i):
   n = length(H)
   if 2i > n then          i is a leaf – nowhere to go
      Terminate with H unchanged
   else if 2i < n then
      left=2i and right=2i+1
      j be index that minimizes
            key[H[left]] and key[[H[right]]
   else if 2i = n then
      j=2i

   if key[H[j]] < key[H[i]] then
      swap array entries H[i] and H[j]
      Heapify-down(H, j)
```


Practice: Heapify-Down

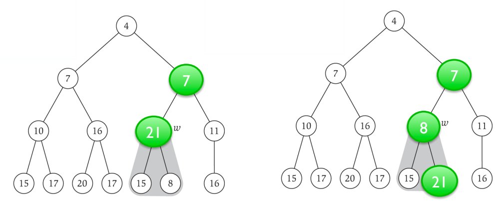
Moved 21 to where element was removed



Practice: Heapify-Down



Practice: Heapify-Down



Runtime of Heapify-Down?

```
Heapify-down(H, i):
   n = length(H)
   if 2i > n then
      Terminate with H unchanged
   else if 2i < n then
      left=2i and right=2i+1
      j be index that minimizes      O(1)
            key[H[left]] and key[[H[right]]
   else if 2i = n then
      j=2i

   if key[H[j]] < key[H[i]] then
      swap array entries H[i] and H[j] O(1)
      Heapify-down(H, j)
```

Num swaps: O(log n)

Implementing Priority Queues with Heaps

Operation	Description	Run Time
StartHeap(N)	Creates an empty heap that can hold N elements	
Insert(v)	Inserts item v into heap	
FindMin()	Identifies minimum element in heap but does not remove it	
Delete(i)	Deletes element in heap at position i	
ExtractMin()	Identifies and deletes an element with minimum key from heap	

Implementing Priority Queues with Heaps

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StartHeap(N)	Creates an empty heap that can hold N elements	O(N)
Insert(v)	Inserts item v into heap	O(log n)
FindMin()	Identifies minimum element in heap but does not remove it	O(1)
Delete(i)	Deletes element in heap at position i	O(log n)
ExtractMin()	Identifies and deletes an element with minimum key from heap	O(log n)

Putting It All Together…

1. Add elements into PQ with the number's value as its priority
2. Then extract the smallest number until done
   - Come out in sorted order

What is the running time of sorting numbers using a PQ implemented with a Heap?

O(n log n)

Comparing Data Structures

Operation	Heap	Unsorted List	Sorted List
Start(N)			
Insert(v)			
FindMin()			
Delete(i)			
ExtractMin()			

Comparing Data Structures

Operation	Heap	Unsorted List	Sorted List
Start(N)	O(N)		
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Delete(i)	O(log n)		
ExtractMin()	O(log n)		

Comparing Data Structures

Operation	Heap	Unsorted List	Sorted List
Start(N)	O(N)	O(1)	O(1)
Insert(v)	O(log n)	O(1)	O(n)
FindMin()	O(1)	O(1)	O(1)
Delete(i)	O(log n)	O(n)	O(1)
ExtractMin()	O(log n)	O(n)	O(1)

Additional Heap Operations

- Access elements in PQ by "name"

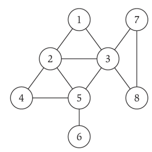
Key	2	4	5	6	9	20	← Priority
Value	3542	5143	8712	1264	9123	5954	← Process id

  - Maintain additional array `Position` that stores current position of each element in heap
- Operations:
  - Delete(Position[v])
    - Does not increase overall running time
  - ChangeKey(v, α)
    - Changes key of element v to α
    - Identify position of element v in array (`Position` array)
    - Change key, heapify

GRAPHS

Undirected Graphs G = (V, E)

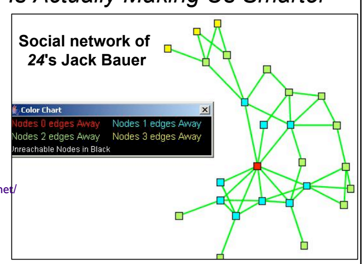
- V = nodes (vertices)
- E = edges between pairs of nodes
- Captures pairwise relationship between objects
- Graph size parameters: n = |V|, m = |E|



V = { 1, 2, 3, 4, 5, 6, 7, 8 }
E = { 1-2, 1-3, 2-3, 2-4, 2-5, 3-5, 3-7, 3-8, 4-5, 5-6 }
n = 8
m = 11

Social Networks

- Node: people; Edge: relationship between 2 people
- *Everything Bad Is Good for You: How Today's Popular Culture Is Actually Making Us Smarter*
  - Television shows have complex plots, complex social networks



http://www.cs.duke.edu/csed/harambeenet/modules.html

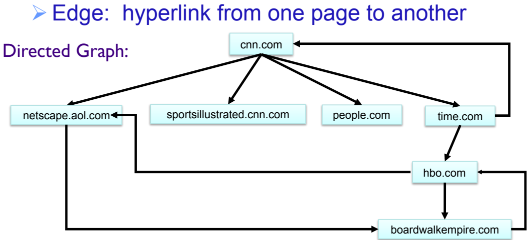
Facebook: Visualizing Friends

http://www.facebook.com/notes/facebook-engineering/visualizing-friendships/469716398919

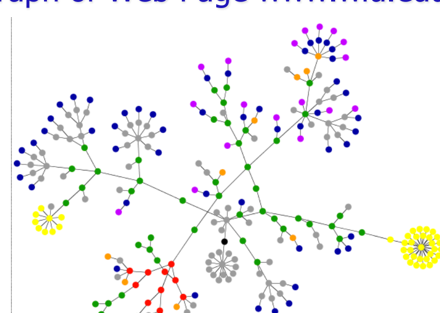
World Wide Web

- Web graph
  - Node: web page
  - Edge: hyperlink from one page to another

Directed Graph:



Graph of Web Page www.wlu.edu

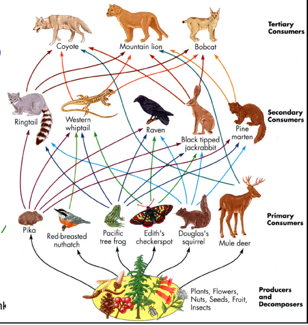


http://www.aharef.info/static/htmlgraph

Ecological Food Web

- Food web graph
  - Node = species
  - Edge = from prey to predator

Directed Graph:



Reference:
`https://www.msu.edu/course/isb/202/ebertmay/images/foodweb.jpg`

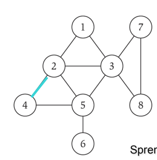
Rock Paper Scissors Lizard Spock

Graph Applications

Graph	Nodes	Edges
transportation	street intersections	highways
communication	computers	fiber optic cables
World Wide Web	web pages	hyperlinks
social	people	relationships
food web	species	predator-prey
software systems	functions	function calls
scheduling	tasks	precedence constraints
circuits	gates	wires

Graph Representation: Adjacency Matrix

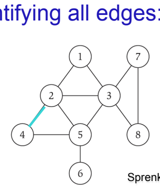
- n×n matrix with A_{uv} = 1 if (u, v) is an edge
  - Two representations of each edge (symmetric matrix)
  - Space?
  - Checking if (u, v) is an edge?
  - Identifying all edges?



	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	0	0
2	1	0	1	1	1	0	0	0
3	1	1	0	0	1	0	1	1
4	0	1	0	1	1	0	0	0
5	0	1	1	1	0	1	0	0
6	0	0	0	0	1	0	0	0
7	0	0	1	0	0	0	0	1
8	0	0	1	0	0	0	1	0

Graph Representation: Adjacency Matrix

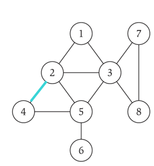
- n×n matrix with A_{uv} = 1 if (u, v) is an edge
  - Two representations of each edge (symmetric matrix)
  - Space: $\Theta(n^2)$
  - Checking if (u, v) is an edge: $\Theta(1)$ time
  - Identifying all edges: $\Theta(n^2)$ time

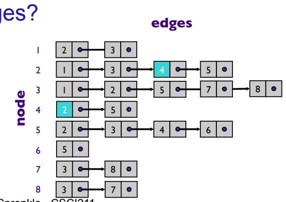


	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	0	0
2	1	0	1	1	1	0	0	0
3	1	1	0	0	1	0	1	1
4	0	1	0	1	1	0	0	0
5	0	1	1	1	0	1	0	0
6	0	0	0	0	1	0	0	0
7	0	0	1	0	0	0	0	1
8	0	0	1	0	0	0	1	0

Graph Representation: Adjacency List

- Node indexed array of lists
  - Two representations of each edge
  - Space? ← What are the extremes?
  - Checking if (u, v) is an edge?
  - Identifying all edges?

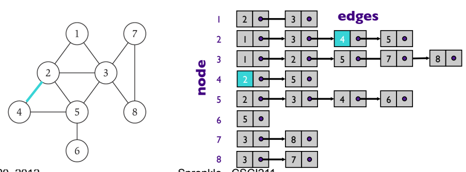




Graph Representation: Adjacency List

- Node indexed array of lists
  - Two representations of each edge
  - Space = 2m + n = O(m + n)
  - Checking if (u, v) is an edge takes O(deg(u)) time
  - Identifying all edges takes Θ(m + n) time

degree = number of neighbors of u



Assignments

- Journals: Finish Chapter 2 for Tuesday
  - Chapter 3 started today but we'll leave it for next week
- Problem Set 2 due Friday