CSCI211: Exam 1

Due Friday, Feb 12 at 5 p.m.

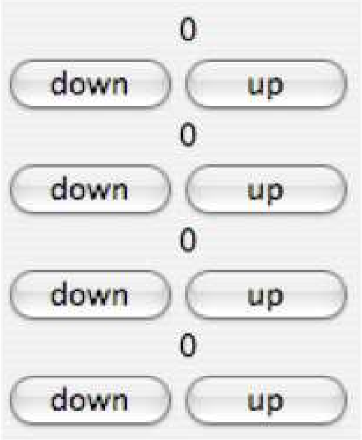
1. **20 pts. Take the following list of functions and arrange them in ascending order of growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$. Justify your answer.**

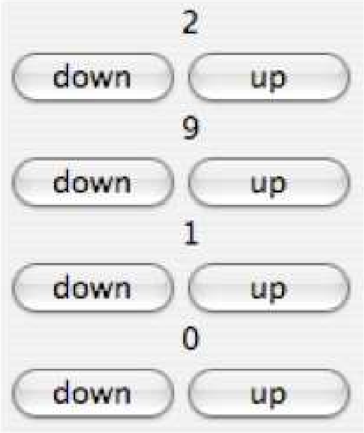
$$\begin{array}{ll} f_1(n) = 2^n & f_2(n) = 2^{2^n} \\ f_3(n) = n! & f_4(n) = nlogn \\ f_5(n) = n^3 & f_6(n) = logn \\ f_7(n) = \sqrt{n} & f_8(n) = n^{logn} \end{array}$$

2. **15 pts. Design an algorithm for checking whether two given words are anagrams, i.e., whether one word can be obtained by permuting the letters of the other. For example, the words *tea* and *eat* are anagrams. Analyze the efficiency of your algorithm.**

3. **20 pts. Imagine there is a program that displays 4 fields. Each field can display a number from 0 to 9. Each field also has an up and down button. Clicking the up button increases the digit displayed, wrapping around from 9 to 0. Clicking the down button decreases the value displayed, wrapping around from 0 to 9.**

   One possible configuration is shown below on the left. The configuration below on the right can be reached by clicking the top up button twice, the second down button once, and the third up button once.





The problem: given a starting value for each field (i.e., a start configuration), a target value for each field (i.e., a target configuration), and a set of forbidden configurations, find the minimum number of button presses required to go from the start configuration to the target configuration without passing through any forbidden values. Design an efficient algorithm to solve this problem and analyze your algorithm's efficiency.

It is not necessary to output the list of buttons clicked, only the number of buttons clicked. Here are some examples, but note that your algorithm should work on all possible values, not just on these specific examples.

Example 1:

Start: 8056
End: 6508
Forbidden: 8057, 8047, 5508, 7508, 6408
Answer: 14

Example 2:

Start: 0000
End: 0002
Forbidden: 0001
Answer: 4

Example 3:

Start: 0000
End: 0002
Forbidden: 0001, 0009, 0010, 0090, 0100, 0900, 1000, 9000
Answer: There is no path

4. **25 pts. President Ruscio is throwing a fund raising party and is deciding whom to call. He has n people from which to choose. He has made up an exhaustive list of pairs of people that know each other. He wants to invite as many people as possible to maximize the donations. However, he has two constraints: at the party, each person should have at least five other people whom they know and five other people whom they don't know. Concisely describe an efficient algorithm that takes as input a list of n people and the list of pairs of people that know each other and outputs the best choice of party invitees (where "best choice" means "has the most invitees"). Analyze your algorithm's efficiency.**

   Hint: Think of the input as a complete undirected graph where an edge (u, v) is colored red if u and v know each other and colored blue if u and v do not know each other. The goal is to find the largest subgraph where each node has at least 5 red edges and at least 5 blue edges.