CSCI211: Problem Set 1

Due Friday, January 21
Points Possible: 25

1. **5 pts. In a room with n people ($n > 2$), every person shakes hands once with every other person. Prove that there are $\frac{n^2-n}{2}$ handshakes.**

2. **5 pts. (1.2) Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.**

   True or false? Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m,w) belongs to S.

3. **6 pts. (1.5) Do problem 5 in Chapter 1 of the text. If your answer is that an algorithm exists, you need to also prove that the algorithm guarantees that it produces a matching that contains no instability.**

4. **4 pts. (2.1-8, CLR) We can extend the O notation to the case of two parameters n and m that can go to infinity independently at different rates. For a given function $g(n, m)$, we denote $O(g(n, m))$ as the set of functions**

   $O(g(n, m)) = \{f(n, m)$: there exist positive constants c, n_0, m_0 such that $0 \leq f(n, m) \leq cg(n, m)$ for all $n \geq n_0, m \geq m_0$ $\}$

   Give corresponding definitions for $\Omega(g(n, m))$ and $\Theta(g(n, m))$.

5. **5 pts. (2.3) Take the following list of functions and arrange them in ascending order of growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$.**

$$\begin{array}{ll} f_1(n) = n^{2.5} & f_2(n) = \sqrt{2n} \\ f_3(n) = n + 10 & f_4(n) = 10^n \\ f_5(n) = 100^n & f_6(n) = n^2 logn \end{array}$$