# CSCI211: Problem Set 1

Due before class Friday, January 18
Points Possible: 25

1. **5 pts. The Fibonacci number sequence is defined as follows:**

$$\begin{aligned} F_0 &= 0 \\ F_1 &= 1 \\ F_i &= F_{i-1} + F_{i-2} \ for \ i \geq 2 \end{aligned}$$

   **The Lucas numbers are defined similarly to the Fibonacci numbers, but have different base conditions. That is**

$$\begin{aligned} L_0 &= 2 \\ L_1 &= 1 \\ L_i &= L_{i-1} + L_{i-2} \ for \ i \geq 2 \end{aligned}$$

   **Show, using induction, that** $L_n = F_{n-1} + F_{n+1} \ for \ n \geq 1.$

2. **4 pts. (1.2) Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.**

   *True or false? Consider an instance of the Stable Matching Problem in which there exists a man $m$ and a woman $w$ such that $m$ is ranked first on the preference list of $w$ and $w$ is ranked first on the preference list of $m$. Then in every stable matching S for this instance, the pair (m,w) belongs to $S$.*

3. **7 pts. (1.5) Do problem 5 in Chapter 1 of the text. If your answer is that an algorithm exists, you need to also prove that the algorithm guarantees that it produces a matching that contains no instability.**

4. **4 pts. (2.1-8, CLR) We can extend the $O$ notation to the case of two parameters $n$ and $m$ that can go to infinity independently at different rates. For a given function $g(n,m)$, we denote $O(g(n,m))$ as the set of functions**

   $O(g(n,m)) = \{f(n,m)$: there exist positive constants $c, n_0, m_0$ such that $0 \leq f(n,m) \leq cg(n,m)$ for all $n \geq n_0, m \geq m_0$ $\}$

   **Give corresponding definitions for $\Omega(g(n,m))$ and $\Theta(g(n,m))$.**

5. **5 pts. (2.3) Take the following list of functions and arrange them in ascending order of growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$.**

$$\begin{array}{ll} f_1(n) = n^{2.5} & f_2(n) = \sqrt{2n} \\ f_3(n) = n + 10 & f_4(n) = 10^n \\ f_5(n) = 100^n & f_6(n) = n^2 logn \end{array}$$