Objectives

- Data structure: Heaps
- Implementing a Priority Queue

- Check in on problem set
- Algorithm runtimes

Moving from integers, lists, arrays

MORE COMPLEX DATA STRUCTURES

Improving Running Times

After overcoming higher-level obstacles, lower-level **implementation details** can **improve runtime**.

PRIORITY QUEUES

Priority Queues

- Elements have a ***priority*** or *key*
- Each time select an element from the priority queue, want the one with *highest* priority
- More formally…
  - Maintains a set of elements S
    - Each element $v \in S$ has a key(v) for its priority
      - Smaller keys represent higher priorities
  - Example methods:
    - Add, delete elements
    - Select element with smallest key

Key	2	4	5	6	9	20	← Priority
Value	3542	5143	8712	1264	9123	5954	← Process id

 Not implementation, just how to envision

Motivating Example: Scheduling Processes

Key	2	4	5	6	9	20	← Priority
Value	3542	5143	8712	1264	9123	5954	← Process id

- Each process has a priority or urgency
- Processes do not arrive in priority order
- **Goal**: run process with highest priority

Using a Priority Queue

- Given API:
  - Add an element with a given key (i.e., priority)
  - Delete an element with a given priority
  - Select element with smallest key/highest priority
  - Get the number of elements in PQ

How could we use a PQ to sort a list of numbers?

Priority Queues for Sorting

1. Add elements into PQ with the number's value as its priority
2. Then extract the smallest number *until* done
   - Come out in sorted order

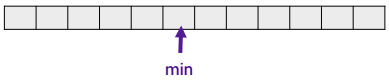
Sorting *n* numbers takes O(n logn) time

What is the goal running time for our PQ's operations? **O(logn)**

Already know our "loops" will be O(n)

Implementing a Priority Queue

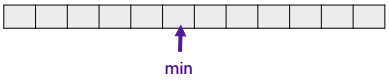
- Consider an *unordered* list, where there is a pointer to minimum



- How difficult (i.e., expensive) is
  - Adding new elements?
  - Extraction?

Implementing a Priority Queue

- Consider an *unordered* list, where there is a pointer to minimum

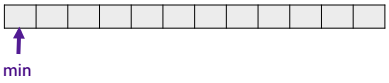


- How difficult (i.e., expensive) is
  - Adding new elements? *easy (O(1))*
  - Extraction? *difficult*
    - Need to find "new" minimum: O(n)

What is the running time for sorting using the PQ in this case? $O(n^2)$

Implementing a Priority Queue

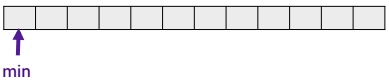
- Consider a *sorted* list where min is at the beginning



- Should you use an array or linked list?
- How difficult is
  - Adding new elements?
  - Extraction?

Implementing a Priority Queue

- Consider a sorted list where min is at the beginning



- Should you use an array or linked list?
- How difficult is
  - Adding new elements? *difficult (insertion)*
  - Extraction? *Easy*

What is the running time for sorting using the PQ in this case? $O(n^2)$

Comparing Data Structures

Operation	Unsorted List	Sorted List
Start(N)		
Insert(v)		
FindMin()		
Delete(i)		
ExtractMin()		

Comparing Data Structures

Operation	Unsorted List	Sorted List
Start(N)	O(1)	O(1)
Insert(v)	O(1)	O(n)
FindMin()	O(1)	O(1)
Delete(i)	O(n)	O(1)
ExtractMin()	O(n)	O(1)

Reflection

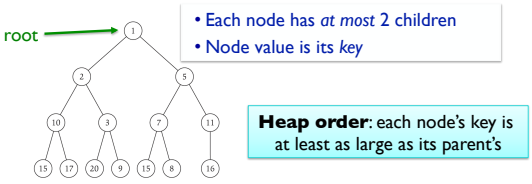
- All of "known" data structures has one operation that takes O(n) time
- Cannot implement PQs with "known" data structures arrays and lists to meet desired O(n log n) runtime

Motivates use of a new data structure (***heap***) to implement PQ

HEAPS

Heap Defined

- Combines benefits of sorted array and list
- Balanced binary tree

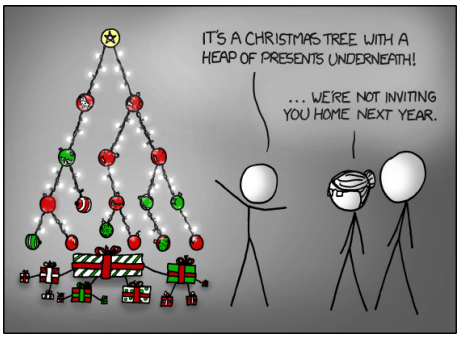


- Each node has *at most* 2 children
- Node value is its *key*

Heap order: each node's key is at least as large as its parent's

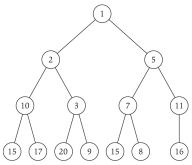
Note: **not** a binary search tree

Heaps



Implementing a Heap

- Option 1: Use pointers
  - Each node keeps
    - Element it stores (key)
    - 3 pointers: 2 children, parent
- Option 2: No pointers
  - Requires knowing upper bound on *n*
  - For node at position *i*
    - left child is at *2i*
    - right child is at *2i+1*



Where does the index in the array start?
If know child's position, what is the position of parent?

Implementing a Heap: Operations

- Finding the minimal element?

Implementing a Heap: Operations

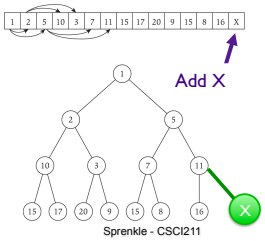
- Finding the minimal element
  - First element
  - O(1)

Implementing a Heap: Operations

- Adding an element?
  - Assume heap has less than N elements

Implementing a Heap: Operations

- Adding an element?
  - Could add element to last position
    - What are possible scenarios?



Implementing a Heap: Operations

- Adding an element?
  - Could add element to last position
    - What are possible scenarios?
      - Heap is no longer balanced
      - Something that is almost a heap but a little off
      - Need `Heapify-up` procedure to fix our heap

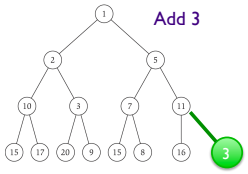
Heapify-Up

Heap — Position where node added

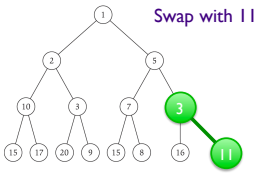
```
Heapify-up(H, i):
   if i > 1 then
       j=parent(i)=floor(i/2)
       if key[H[i]] < key[H[j]] then
          swap array entries H[i] and H[j]
          Heapify-up(H, j)
```

- Why does this algorithm work?
- What is the intuition?

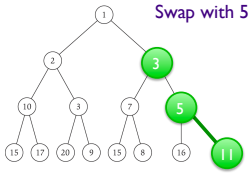
Practice: Heapify-Up



Practice: Heapify-Up



Practice: Heapify-Up



Heapify-Up

- Claim. Assuming array H is almost a heap with key of H[i] too small, Heapify-Up fixes the heap property in O(log i) time
  - Can insert a new element in a heap of *n* elements in O(log n) time

Assignments

- Journals: Finish Chapter 2 for Tuesday
- Problem Set 2 due Friday