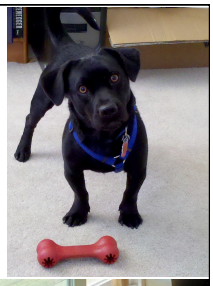
## CSCI211: Intro Objectives

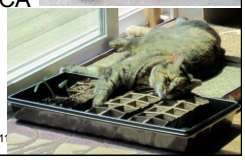
- Introduction to Algorithms, Analysis
- Course summary
- Reviewing proof techniques



## My Bio

- From Dallastown, PA
- B.S., Gettysburg College
- M.S., Duke University
- Ph.D., University of Delaware
- For fun: pop culture, ultimate, gardening
- Volunteer at Rockbridge SPCA







## What This Course Is About


From *30 Rock*



> Now, everything comes down to expert knowledge of **algorithms** and **data structures**.
> If you don't speak fluent **O-notation**, you may have trouble getting your next job at the technology companies in the forefront.
> -- Larry Freeman

> For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense, and even mysterious. But once unlocked, they cast a **brilliant new light** on some aspect of computing.
> -- *Francis Sullivan*



## Motivation Google

- From a Google interview preparation email

> Get your algorithms straight (they may comprise up to a **third** of your interview).
> Visit: http://en.wikipedia.org/wiki/List_of_algorithm_general_topics
> and examine this list of algorithms:
> http://en.wikipedia.org/wiki/List_of_algorithms
> and data structures: http://en.wikipedia.org/wiki/List_of_data_structures
> Write out all the algorithms yourself from start to finish and make sure they're working.



## What is an Algorithm?

- Precise procedure to solve a problem
- Completes in a finite number of steps

## Questions to Consider

- What are our goals when designing algorithms?
- How do we know when we've met our goals?

- Goals: Correctness, Efficiency
- Use analysis to show/prove



## Course Goals

- Learn how to formulate precise problem descriptions
- Learn specific algorithm design techniques and how to apply them
- Learn how to analyze algorithms for efficiency and for correctness
- Learn when no exact, efficient solution is possible



## Course Content

- Algorithm analysis
  - Formal – proofs; Asymptotic bounds
- Advanced data structures
  - e.g., heaps, graphs
- Greedy Algorithms
- Dynamic Programming
- Divide and Conquer
- Network Flow
- Computational Intractability



## Course Notes

- Textbook: *Algorithm Design*
  - Optional: CLRS
- Participation is encouraged
  - Individual, group, class
- Assignments:
  - Reading text, writing brief summaries
    - Readings through Friday due following Tuesday
  - Solutions to problems
  - Analysis of solutions
  - Programming (little)

  } Given on Friday, due next Friday



## Course Grading

- 40% Individual written and programming homework assignments
- 30% Two midterm exams
- 20% Final
- 5% Text book reading summaries, weekly
  - In a journal on wiki
- 5% Participation and attendance



## Journal Content

- Brief summary of chapter/section
  - ~1 paragraph of about 5-10 sentences/section; feel free to write more if that will help you
- Include motivations for the given problem, as appropriate
- For algorithms, brief sketch of algorithm, intuition, and implementation
  - Include runtime
- Questions you have about motivation/solution/proofs/ analysis
- Discuss anything that makes more sense after reading it again, after it was presented in class (or vice versa)
- Anything that you want to remember, anything that will help you
- Say something about how readable/interesting the section was on scale of 1 to 10

## ALGORITHMS



## Computational Problem Solving 101

- Computational Problem
  - A problem that can be solved by logic
- To solve the problem:
  1. Create a *model* of the problem
  2. Design an *algorithm* for solving the problem using the model
  3. Write a *program* that implements the algorithm



## Computational Problem Solving 101

- Algorithm: a well-defined recipe for solving a problem
  - Has a finite number of steps
  - Completes in a finite amount of time
- Program
  - An algorithm written in a programming language
  - Important to consider implementation's effect on runtime



## PROOFS



## Why Proofs?

- What are insufficient alternatives?
- How can we prove something isn't true?



## Why Proofs?

- What are insufficient alternatives?
  - Examples
    - Considered all possible?
  - Empirical/statistical evidence
    - Ex: "Lying" with statistics
- How can we prove something isn't true?
  - One counterexample

Need irrefutable proof that something is true—for ***all*** possibilities

## Soap Opera Proofs

- "It's the only thing that makes sense."



## Analyzing Statistics

From Joel Feinstein
University of Nottingham
"Why do we do proofs"

Two hospitals (A and B) each claim to be better at treating a certain disease than the other.

**Hospital A**

- cured a greater % of its *male patients* last year than Hospital B
- cured a greater % of its *female patients* last year than Hospital B

**Hospital B**

- cured a greater % of its *patients* last year than Hospital A

Given that none of the #s involved are zero, is it possible that both hospitals have their calculations correct?
If so, which hospital would you rather be treated by?



## Example

From Joel Feinstein
University of Nottingham
"Why do we do proofs"

| Hospital | Male Patients | % | Female Patients | % | Total Patients | % |
|---|---|---|---|---|---|---|
| A | 50/100 | 50% | 1/1 | 100% | 51/101 | 50.5% |
| B | 24/50 | 48% | 49/50 | 98% | 73/100 | 73% |

Well-known phenomenon: Simpson's Paradox



## Common Types of Proofs?



## Common Types of Proofs

- Direct proofs
  - Series of true statements, each implies the next
- Proof by contradiction
- Proof by induction



## Proof By Contradiction

What are the steps to a proof by contradiction?

## Proof By Contradiction

1. Assume the proposition (P) we want to prove is false
2. Reason to a contradiction
3. Conclude that P must therefore be true



## Prove: There are Infinitely Many Primes

- What is a prime number?
- What is not-a-prime number?

- What is our first step (proof by contradiction)?
- What do we want to show?



## Prove: There are Infinitely Many Primes

- Assume there are a finite number of prime numbers
  - List them: $p_1, p_2 ..., p_n$
- Consider the number $q = p_1p_2... p_n + 1$

What are the possibilities for q?

q is either composite or prime



## Prove: There are Infinitely Many Primes

- Assume there are a finite number of prime numbers
  - List them: $p_1, p_2 ..., p_n$
- Consider the number $q = p_1p_2... p_n + 1$
- Case: q is composite
  - If we divide q by any of the primes, we get a remainder of 1 → q is not composite



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- Consider the number $q = p_1p_2... p_n + 1$
- Case: q is composite
  - If we divide q by any of the primes, we get a remainder of 1 → q is not composite
- Therefore, q is prime, but q is larger than any of the finitely enumerated prime numbers listed → *Contradiction*

Proof thanks to Euclid



## Proof By Induction

What are the steps to a proof by induction?

## Proof By Induction

1. What you want to prove
2. Base case
   - Typical: Show statement holds for $n = 0$ or $n = 1$
3. ***Induction hypothesis***
4. Induction step: show that adding one to n also holds true
   - Relies on earlier assumptions

When/why is induction useful?

Show true for all (infinite) possibilities
Show works for "one more"



## Proof By Induction

1. State your P(n).
   - P(n) is a property as a function of *n*
     - State for which *n* you will prove your P(n) to be true
2. State your base case.
   - State for which n your base case is true, and prove it
     - Use the smallest *n* for which your statement is true
3. State your induction hypothesis
   - Without an induction hypothesis, the proof falls apart.
   - Usually it is just restating your P(n), with no restriction on *n* (an arbitrary n)
4. Inductive Step.
   - Consider P(n + 1).
     - Try to prove a larger case of the problem than you assumed in your induction hypothesis.
   - Keep in mind: What are you trying to prove?
   - Use your induction hypothesis, and clearly state where it is used. If you haven't used your induction hypothesis, then you are not doing a proof by induction.
5. Conclusion.
   - Optionally, restate the problem.



## Example of Induction Proof

**Prove**:
2+4+6+8+… + 2n = n*(n+1)

For what values of *n* do we want to prove this is true?

A: where *n* is a natural number



## Example of Induction Proof

**Prove**: 2+4+6+8+… + 2n = n*(n+1)

(where *n* is a natural number)

- **Base case**: $n = 1$ →
  - 2*1 = 1*(1+1) ✔



## Example of Induction Proof

**Prove**: 2+4+6+8+… + 2n = n*(n+1)

(where *n* is a natural number)

- **Base case**: $n = 1$ →
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- Induction Hypothesis:
  - Assume statement is true for some arbitrary k > 1



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- **Base case**: *n* = 1 →
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- Induction Hypothesis:
  - Assume statement is true for some arbitrary k > 1
- Prove holds for *k*+1, i.e., show that 2+4+6+8+… + 2k + 2(k+1) = (k+1)*((k+1)+1)



**Prove**: 2+4+6+8+… + 2n = n*(n+1)

- **Base case**: *n* = 1 → 2*1 = 1*(1+1) ✔
- Assume statement is true for arbitrary *n*=*k*>1
- Prove true for *k*+1, i.e., show that 2+4+6+8+… + 2k + 2(k+1) = (k+1)*((k+1)+1)
  - 2+4+6+8+… + 2k + 2(k+1)
  - = k*(k+1) + 2(k+1)
  - = $k^2$ + k + 2k + 1
  - = $k^2$ + 3k + 1
  - = (k+1)*(k+2)
  - = (k+1)*((k+1)+1) ✔

Approach shown: transform LHS to RHS

I want to see these steps in your proofs!



## Looking Ahead

- Check out course wiki page
  - Test username/password
  - Decide which style of journal you want: wiki or blog
- Read first two pages of book's preface, Chapter 1 of book
  - Summarize on Wiki by next Tuesday @ midnight